Preface

As we enter the twenty-first century, techniques borrowed from equilibrium and non-equilibrium statistical physics have become widely applied to disciplines never imagined by their founders. Statistical physics is turning into an essential discipline and a fundamental framework for understanding and making quantitative predictions on diverse phenomena involving a large number of interacting degrees of freedom. These degrees of freedom may represent fundamental particles, such as electrons or quarks, or neurons carrying information through synapses, or even speculative agents trading in a competitive financial market. This holistic precept, that the whole is not necessarily equal to the sum of its parts, finds in statistical physics its most beloved tool.

Phase transitions and critical phenomena have consistently been among the principal subjects of active studies in statistical physics. The simple act of transforming one state of matter or phase into another, for instance by changing the temperature, has always captivated the curious mind. In that way one can convert an almost uninteresting state of matter into a superconducting material with tremendous implications and applications. The Large Hadron Collider at the European Organization for Nuclear Research (CERN), which is currently exploring the nature of fundamental interactions at high energies, relies on the use of superconducting magnets, electromagnets built out of coils of superconducting niobium-tin wire cooled by liquid Helium. Those magnets not only consume less power but most importantly can achieve an order of magnitude stronger fields than ordinary magnets, a fact that is crucial to reach such high energies.

The unusual set of physical properties known today as critical phenomena were discovered and apparently first reported in the Annales de Chimie et de Physique (1822-1823) by the Baron Charles Cagniard de la Tour. He performed experiments on liquids (water, alcohol and ether) sealed in a glass cell under pressure, and observed the remarkable fact that above a certain temperature, that itself depends on the particular substance, the surface tension between the liquid and vapour disappeared, thus discovering what is known today as the supercritical fluid phase. Trying to prove that beyond a certain temperature the liquid gasifies regardless of pressure, he also noticed that near particular pressure and temperature values something unusual happened. In the neighbourhood of this point, known as the critical point, the liquid becomes increasingly milky, indicating that visible light is being strongly scattered. The term critical point was coined later in 1869 by Thomas Andrews who observed that carbon dioxide at 31 degree Celsius and 73 atmospheres pressure displayed the phenomenon of critical opalescence, that turbid and milky state previously observed by Cagniard de la Tour in other substances. The underlying universality of critical phenomena escaped the attention of their founders. It was Pierre Curie around 1895 who realized about the similarity between the critical behaviours of a liquid-gas phase transition and that of the ferromagnetic transition in iron. The formal connection and derived analogies between unrelated physical materials behaving in a similar, universal, way near a continuous phase transition constitutes one of the landmarks of critical phenomena. Since the discovery of the renormalization group method in the early 1970s, the realm of applications of the concepts of scale invariance and criticality has pervaded several fields in the natural and social sciences. Thus, in perspective, it is of no surprise that these concepts, and the methods used to study them, can be applied to disciplines as diverse as the ones indicated in our introductory paragraph.

This book provides an introductory account on the theory of phase transitions and critical phenomena. The basic knowledge of the theory of phase transitions and critical phenomena is now recognized to be indispensable for students and researchers from many fields of physics and related disciplines. The book has been written having in mind an advanced undergraduate or graduate student in science or mathematics. It has been assumed that the reader has finished introductory courses of statistical mechanics in addition to elementary courses in calculus, Fourier analysis, and probability theory. Very basic undergraduate knowledge of quantum mechanics is required to understand the very few extensions of the classical theory. Clarity and detailed user-friendly derivations of usually accepted, as elementary and not so elementary, concepts have been our guiding principle. We preferred this style of presentation to what is sometimes known as rigorous, where at the expense of making the argument so sharp one looses track of the main idea.

One of our goals in writing this book is to provide the mathematical tools necessary for the students to compute properties of critical systems in diverse contexts and disciplines, such as biophysics or complex systems. Almost all parts are written in a self-contained manner and all new concepts and calculations are explained in much detail without assuming prior knowledge of phase transitions and critical phenomena. We have avoided historical presentations of various topics allowing us compact derivations of the concepts without hiding details. For example, it is typical to first introduce the scaling hypothesis and then the renormalization group method as a way of justifying that hypothesis. Rather, we preferred to derive the scaling laws directly once the concept of a renormalization transformation is introduced which, in our opinion, is a more natural and pedagogical way of presenting the material.

Another of the goals of this book is to prepare the reader to start reading more advanced books and research papers, in which basic accounts of common knowledge are often omitted and consequently beginners are trapped in the jungle of undefined jargons and complicated manipulations. Serious attempts have been directed toward a self-contained modular approach so that the reader does not have to refer to other sources for supplementary information. Accordingly, most of the concepts and calculations are described in detail, sometimes with additional/auxiliary descriptions given in appendices and exercises. It is, of course, impossible to cover all the topics related to phase transitions and critical phenomena in a single volume of this introductory nature. One main omission is the general subject of quantum phase transitions, which happen at zero temperature as a result of changes in the parameters of the Hamiltonian representing the physical system. Although by itself a topic for a second volume, we have explained a few extensions of classical concepts to the quantum realm when appropriate and not in danger of jeopardizing the main ideas. Most of these extensions are written in the appendices. The bibliography at the end of the book will guide the reader to other topics uncovered in this book and also to more advanced references.

A number of important concepts and methods have been developed, such as mean-field theory, scaling theory, the renormalization group method, exact solutions, series expansions, and Monte Carlo simulations, most of which have turned out to be valuable tools not only in statistical physics but also in other fields of physics. The present book also contains pedagogical presentations of statistical field theory methods, including a chapter on conformal field theory, random systems, percolation, the important use of dualities, and various modern developments hard to find in a single textbook on phase transitions. Moreover, as mentioned above, a series of appendices expand and clarify several issues not developed in the main text. It has been done in this way to avoid getting stuck in details and thereby losing the main flow of ideas. We would like to invite the reader, however, to seriously explore those appendices in a second reading since they are very useful to understand the depth and extensions of a particular topic.

In the first half of this book, standard topics such as mean-field theory, the renormalization group, and statistical field theory methods are explained. Then, slightly more advanced, but commonly encountered, concepts and methods follow, including the conformal field theory, the Kosterlitz-Thouless transition, effects of randomness, exact solutions, duality, and numerical techniques. Special emphasis has been laid on providing a physically intuitive description, sometimes with certain sacrifice of mathematical rigor, except in the chapters that discuss exact solutions and duality. The first five chapters are very basic and quintessential, followed by several chapters that can be read independently of each other provided that the first five chapters have been finished. The important role played by symmetry and topology in understanding the competition between phases and the resulting emergent collective behaviour, giving rise to rigidity and soft elementary excitations, is stressed throughout the book. Most importantly, in accordance with Sophocles' advice,[1] exercises are presented as the topics develop with solutions found at the end of the book, thus giving the text a self-learning character. It is strongly recommended to solve (at least try to solve) the exercises as one proceeds in reading, since they often contain vital information to understand the logic developed in the main text.

The book reflects lectures given by the authors at their Universities to graduate students on the same topics and is thus classroom-tested for its usefulness for beginners to this field. Students attending those courses contributed significantly to the improvement of presentation and material selection and the authors are very grateful to them. We would like to express our special thanks to Matthew Dean Jones and Zsolt Bertalan for proofreading and providing insightful remarks. We are also indebted to John Cardy, Pierluigi Contucci, Michael Fisher, Cristian Giardina, Norio Kawakami, Makoto Oka, Andrea Pelissetto and David Sherrington for their crucial suggestions and comments on the draft. Shu Tanaka kindly drew the impressive picture on the cover of this book.

[1] "One learns by doing a thing; for though you think you know it, you have no certainty until you try."

Following the convention of many textbooks, we did not directly refer to original research papers for almost all topics in this book. This never means to claim priority for the materials presented. On the contrary, virtually almost all concepts, methods and conclusions are well-established, standard ones. The book simply reflects the authors' interpretation of what constitutes a concise, consistent, coherent and clear manner of presenting a wide range of topics. Correspondingly, we tried to avoid attributing each result to a specific person except for a limited number of very common names including (but not limited to) the Ising model, Heisenberg model, Landau theory, Virasoro algebra, Kosterlitz-Thouless transition, Sherrington-Kirkpatrick model, and Lee-Yang zeros. The reader is referred to the bibliography at the end of the book for more detailed sources of information on the original references. We nevertheless would like to express our sincere apologies to those who contributed to the developments of the field for leaving out their names with the expectation that our approach is understood and accepted.

We hope this book will help anyone interested in this fascinating subject and, moreover, inspire young scientists to continue developing this profound and far reaching field of science.[2]

Hidetoshi Nishimori and Gerardo Ortiz
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[2]Updates, amendments and addenda will be posted on a dedicated web page at http://mypage.iu.edu/~ortizg/bookEPTCP.htm